

## Mathematical Concepts for Computer Science *Probability Revision.*

We model random events by thinking of some *experiment*, with a set  $\Omega$  of possible *outcomes*, often written as  $\omega \in \Omega$ . An *event* is a subset of  $\Omega$ , say  $A \in \Omega$ . The probability of an event, written  $\mathbb{P}(A)$ , is a number between 0 and 1. You should be able to draw Venn diagrams to represent events. You should be able to calculate with probabilities, including conditional probability and independence. You should know the factorial function and the binomial coefficient.

### Question 0.

(i) If  $A, B, C$  are three events, show that

$$\mathbb{P}(A^c \cap (B \cup C)) = \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(B \cap C) - \mathbb{P}(C \cap A) - \mathbb{P}(A \cap B) + \mathbb{P}(A \cap B \cap C).$$

(ii) How many of the numbers  $1, \dots, 500$  are not divisible by 7 but are divisible by 3 or 5?

**Question 1.** In a group of 10 people what is the probability that no two of them have the same birthday?

**Question 2.** Compute the (approximate) probability that in a room of  $n$  people, at least two have the same birthday.

**Question 3.** I throw a coin 5 times. The throws are independent. In each throw, the probability of heads is  $p$ . What is the probability that exactly two throws yielded heads? What is the probability of two consecutive heads?

### Question 4.

(i) When  $\mathbb{P}(A) = 1/3, \mathbb{P}(B) = 1/2$  and  $\mathbb{P}(A \cup B) = 3/4$ , calculate  $\mathbb{P}(A \cap B)$ . What is the probability that  $A$  occurs or  $B$  occurs but not both?

(ii) When  $\mathbb{P}(A) = 3/4$  and  $\mathbb{P}(B) = 1/3$ , show that  $1/12 \leq \mathbb{P}(A \cap B) \leq 1/3$ , and give examples to show that both extremes are possible. Find corresponding bounds for  $\mathbb{P}(A \cup B)$ .

**Question 5.** You are one in a group of five mafiosi. Each of you has a vendetta against one of the others (chosen at random), and has hired a hitman to kill him. What is the probability that no one has hired a hit man to kill you?

**Question 6.** I draw 4 cards at random from a standard deck of playing cards. What is the probability that I have one of each suit?

**Question 7.** I throw three fair dice, and sum up my throws. Calculate the probability of scoring  $n$ , for all  $3 \leq n \leq 18$ .

**Question 8.** After being tested, you are diagnosed with a disease. You know that 1% of the population has the disease. Write  $D$  for the event “you have the disease” and  $T$  for “the test says you have the disease”. It is known that the test is imperfect:  $\mathbb{P}(T|D) = 0.98$  and  $\mathbb{P}(\text{not } T|\text{not } D) = 0.95$ .

(i) Given that you test positive what is the probability that you truly have the disease?

(ii) You obtain a second opinion, i.e. an independent repetition of the test. You test positive again. Given this, what is the probability that you truly have the disease?

**Question 9.** A family has three children. The probability of giving birth to a boy is 50%. Let  $N$  be the number of boys in the family.

(a) What is the distribution of  $N$ ?

(b) At least one of the children is a boy. Using your answer in part (a), calculate the probability that the other two are boys.

(c) My parents have three children. What is the probability that my two siblings are boys?

Hint: the answer is not the same as the answer to part (b).

**Question 10.** A monkey sits at a typewriter, pressing letters A-Z at random, and composes a string one billion characters long. What is the probability that letters 5 – 12 spell out NETWORKS? What is the expected number of times that NETWORKS appears in the entire string?