Problems on Matrices and Determinants

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1. a. If a matrix has 12 elements, what are the possible orders it can have? What if it has 7 elements?

   b. Is it possible to define the matrix A+B, when

      i. A has 3 rows and B has 2 rows.

      ii. A has 2 columns and B has 4 columns.

      iii. A has 3 rows and B has 2 columns.

      iv. Both A and B are square matrices of the same order.

2. a. If $\begin{bmatrix} x+y & y-z \\ z-2x & y-x \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & -1 \end{bmatrix}$ find x,y,z.

   b. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, find x,y,z,w.

   c. Find the value of x,y,z and a which satisfy the matrix equation

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix} \tag{1}$$

3. If

$$A = \begin{bmatrix} 0 & -1 & 2 \\ 3 & 4 & -5 \end{bmatrix}, B = \begin{bmatrix} -2 & 0 & 3 \\ 4 & -5 & 6 \end{bmatrix}, C = \begin{bmatrix} 4 & -7 & -2 \\ 0 & -5 & 1 \end{bmatrix} \tag{2}$$

   Find 3A-2B+C. Also verify that $(A+B)+C = A+(B+C)$.

4. Evaluate the following :

a. $\begin{bmatrix}1\\2\\3\end{bmatrix}\begin{bmatrix}2 & 3 & 4 & 5\end{bmatrix}$.

b. $\begin{bmatrix}a & b\end{bmatrix}\begin{bmatrix}c\\d\end{bmatrix} + \begin{bmatrix}a & b & c & d\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$.

c. $\begin{bmatrix}1 & -1\\0 & 2\\2 & 3\end{bmatrix} \times \left(\begin{bmatrix}1 & 0 & 2\\2 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1 & 2\\1 & 0 & 2\end{bmatrix} \right)$.

5. Prove that the matrix given by A =

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

satisfies the relation $A^2 - A(a+d) + (ad-bc)I = 0$, where I is a unit matrix of order two.

6. Express the following matrix as the sum of a symmetric and skew symmetric matrix.
$\begin{bmatrix}-1 & 7 & 1\\2 & 3 & 4\\5 & 0 & 5\end{bmatrix}$

7. Show that the elements on the main diagonal of a skew symmetric matrix are all zeros.

8. Find the symmetric and skew symmetric parts of the matrix.

A =
$$\begin{bmatrix} 1 & 2 & 4 \\ 6 & 8 & 1 \\ 3 & 5 & 7 \end{bmatrix}$$

9. Find the determinant

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

10. Find the adjoint of the matrix

A =

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

11. Let A be the matrix

$$\begin{bmatrix} 3 & 8 \\ 2 & 1 \end{bmatrix}$$

Find A^{-1} and verify that $A^{-1} = \frac{1}{13}A - \frac{4}{13}I$, where I is 2×2 unit matrix.

12. If A = $\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}.A^{-1}$.

13. Compute the inverse of the matrix

A =

$$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$$

14. Compute the A^{-1} of the matrix

A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.