CS 603 Mathematical Concepts in Computer Science

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1.

- $E(x) \vee S(x)$
- $P(x) \wedge N(x)$
- $P(x) \wedge G(x)$
- $P(x) \Rightarrow G(x)$
- $P(x) \Rightarrow (Q(x) \wedge \neg E(x))$

2.

- $\neg\forall x(Car(x) \Rightarrow Carb(x))$ $\exists x(Car(x) \wedge \neg Carb(x))$
- $\exists x(r(x) \vee p(x))$ $\neg\forall x \neg(r(x) \vee p(x))$
- $\forall x(dog(x) \Rightarrow \neg intell(x))$ $\neg\exists x(dog(x) \wedge intell(x))$
- $\forall x(baby(x) \Rightarrow ill(x))$ $\neg\exists x \neg(baby(x) \Rightarrow ill(x))$
- $\forall x(N(x) \vee S(x))$
- $\exists x(\neg r(x))$ $\neg\forall x(r(x))$
- $\forall x((connect(x) \wedge cfree(x)) \Rightarrow tree(x))$ $\neg\exists x \neg((connect(x) \wedge cfree(x)) \Rightarrow tree(x))$
- $\neg\forall x(graph(x) \Rightarrow connect(x))$ $\exists x(graph(x) \wedge \neg connect(x))$

3.a)
1. $\forall x(p(x) \vee a(x))$
2.$alive(dog) \wedge \neg p(dog)$
3.$\forall x(a(x) \Rightarrow h(x))$
$- -$
$h(dog)$

Proof From 1, UI, $p(c) \vee a(c)$ implies $\neg p(c) \Rightarrow a(c) - - - - - - 4$
From 1,4 Use hypothetical syllogism, $\neg p(c) \Rightarrow h(c) - - - - - - 5$
From 2, simplification, $\neg p(dog) - - - - 6$
Since c in 4 is arbitrary, from 6,5 EG, we get $h(dog)$

3.b) Add the following statement, there are ignorant people.
1. $\forall x(m(x) \Rightarrow \neg i(x))$
2. $\forall x(i(x) \Rightarrow h(x))$
3. $\exists x i(x)$
$-----------------------------------$
$\exists x(h(x) \wedge \neg m(x))$

Proof From 1, UI, $m(c) \Rightarrow \neg i(c)$ $----------$ 5
From 2, UI, $i(c) \Rightarrow h(c)$ $---------$6
From 3, EI, $i(c)$ $--------$ 7
From 5, $i(c) \Rightarrow \neg m(c)$ $--------$ 8
From 6,7, $h(c)$ $------$9
From 7,8, $\neg m(c)$ $---$ 10
From 9,10, $h(c) \wedge \neg m(c)$ $-----------$11
From 11, EG, $\exists x(h(x) \wedge \neg m(x))$.

3.c)
1. $\forall x(b(x) \Rightarrow ill(x))$
2. $\forall x(croc(x) \Rightarrow \neg disp(x))$
3.$\forall x(ill(x) \Rightarrow disp(x))$
$--------------------------------$
$\forall x(b(x) \Rightarrow \neg croc(x))$

Proof From 1, UI, $b(a) \Rightarrow ill(a)$ $---------$4
From 3, UI, $ill(a) \Rightarrow disp(a)$ $---------$5
From 4,5, Hypothetical syllogism, $b(a) \Rightarrow disp(a)$ $-------$ 6
From 2, UI $croc(a) \Rightarrow \neg disp(a)$ $-----------$7
From 7, $disp(a) \Rightarrow \neg croc(a)$ $--------$ 8
From 6,8 $b(a) \Rightarrow \neg croc(a)$ $------$9
From 9, UG $\forall x(b(x) \Rightarrow \neg croc(x))$

3.d)
1. $ave \Rightarrow arith$
2. $\neg ave \Rightarrow \neg cap$
3.$your \Rightarrow \neg arith$
$-----------------------------$
$your \Rightarrow \neg cap$

Proof From 1, $\neg arith \Rightarrow \neg ave$ $---------$4
From 2,4, $\neg arith \Rightarrow \neg cap$ $------$5
From 3,5, $your \Rightarrow \neg cap$

3.e)
1.$\forall x(i(x) \Rightarrow r(x))$

2. $\exists x(i(x) \wedge pow(x))$

$---------------------------$

$\exists x(r(x) \wedge pow(x))$

Proof From 1, UI, $i(d) \Rightarrow r(d) ----4$
From 2, EI, $i(d) \wedge pow(d) -------- 5$
From 5, simplification, $i(d) ----- 6$
From 5, simplification, $pow(d) -------- 7$
From 4,6, $r(d) ----- 8$
From 7,8, $r(d) \wedge pow(d) ------9$
From 9, EG, $\exists x(r(x) \wedge pow(x))$.

3.f) False. Counter example Figure 1.
3.g)

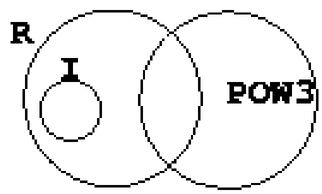

Figure 1: Counter example

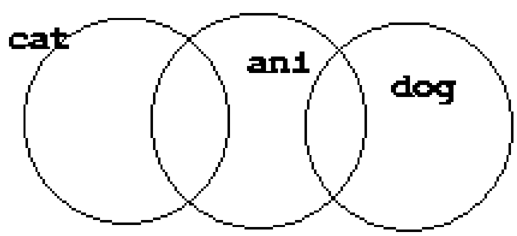

Figure 2: Counter example

1.$\forall x(c(x) \Rightarrow s(x))$
2.$\exists x(e(x) \wedge \neg s(x))$

$-----------------------------$

$\exists x(e(x) \wedge \neg c(x))$

Proof From 1, UI, $c(d) \Rightarrow s(d) ---------3$
From 2, EI, $e(d) \wedge \neg s(d) ------4$
From 3, $\neg s(d) \Rightarrow \neg c(d) ---------5$
From 4, simplification,$\neg s(d) ------------- 6$
From 5,6, $\neg c(d) ---------- 7$
From 4,7, $e(d) \wedge \neg c(d) --------------- 8$
From 8, EG, $\exists x(e(x) \wedge \neg c(x))$

3.h) False. Counter example figure 2.
3.i)
1.$\forall x(d(x) \Rightarrow c(x))$
2.$\exists x(a(x) \wedge d(x))$
$- -$
$\exists x(a(x) \wedge c(x))$

Proof From 1, UI, $d(a) \Rightarrow c(a) - - - -3$
From 2, EI, $a(a) \wedge d(a) - - - - - - - - -4$
From 3, simplification, $d(a) - - - - - - - - -5$
From 3,5, $c(a) - - - - - - - - - -$ 6
From 4, simplification $a(a) - - - - - - - - -7$
From 6,7, EG $\exists x(a(x) \wedge c(x))$

4.a)
1.$\forall x(c(x) \Rightarrow h(x))$
2.$\forall x(s(x) \Rightarrow c(x))$
$- -$
$\forall x(s(x) \Rightarrow h(x))$

Proof From 2, UI, $s(d) \Rightarrow c(d) - - - -3$
From 1,UI, $c(d) \Rightarrow h(d) - - - - - -$ 4
From 3,4, $s(d) \Rightarrow h(d) - - - - - - - -$ 5
From 5, UG, $\forall x(s(x) \Rightarrow h(x))$

4.b)
Counter Example. See figure 3
4.c)

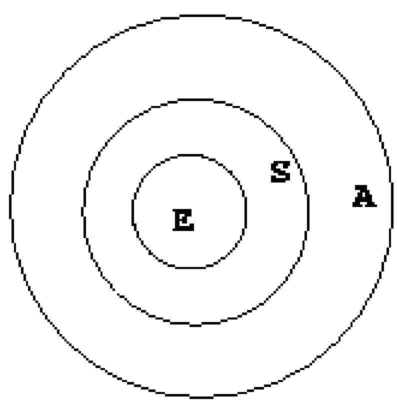

Figure 3: Counter example

1.$\forall x(a(x) \Rightarrow s(x))$
2.$\exists x(a(x) \wedge e(x)$

$- -$
$\exists x(s(x) \wedge s(x))$

Proof From 2, EI, $a(c) \wedge e(c) - - - - - - 3$
From 3, simplification, $a(c) - - - - - 4$
From 1, UI, $a(c) \Rightarrow s(c) - - - - - - 5$
From 4,5, $s(c) - - - - - - - - - - - - - 6$
From 3, simplification, $e(c) - - - - - - - - 7$
From 6,7, $s(c) \wedge e(c) - - - - - - - - - 8$
From 8, EG $\exists x((s(x) \wedge e(x))$

4.d)
1.$\exists x(h(x) \wedge v(x))$
2.$\forall x(h(x) \Rightarrow m(x))$
$- -$
$\exists x(m(x) \wedge v(x))$

Proof From 1, EI, $h(a) \wedge v(a) - - - - 3$
From 2, UI, $h(a) \Rightarrow m(a) - - - - - - - - - 4$
From 3, simplification, $h(a) - - - - - - - - 5$
From 3, simplification, $v(a) - - - - - - - - - 6$
From 4,5, $m(a) - - - - - - - - - - - 7$
From 6,7, $m(a) \wedge v(a) - - - - - - - - - 8$
From 8, EG, $\exists x(m(x) \wedge v(x))$

4.e)
1.$\forall x(mother(x) \Rightarrow \neg male(x))$
2.$\exists x(male(x) \wedge poli(x))$
$- -$
$\exists x(poli(x) \wedge \neg mother(x))$

Proof From 1, UI, $mother(c) \Rightarrow \neg male(c) - - - - - - - - - 3$
From 2, EI, $male(c) \wedge poli(c) - - - - - - - - 4$
From 3, $male(c) \Rightarrow \neg mother(c) - - - - - - 5$
From 4, simplification, $male(c) - - - - - - - - - 6$
From 4, simplification, $poli(c) - - - - - - - - 7$
From 5,6, $\neg mother(c) - - - - - - 8$
From 7,8, $polic(c) \wedge \neg mother(c) - - - - - - - - - 9$
From 9, EG, $\exists x(poli(x) \wedge \neg mother(x))$

4.f)
1.$\exists x(f(x) \wedge \neg m(x))$
2.$\exists x(p(x) \wedge \neg f(x))$
$- -$
$\exists x(p(x) \wedge \neg m(x))$

Proof

4.g)
1.$\forall x(d(x) \Rightarrow grad(x))$
2.$\exists x(d(x) \wedge \neg golf(x))$
$- -$
$\exists x(golf(x) \wedge \neg grad(x))$
The claim is False. See figure 4.
4.h)

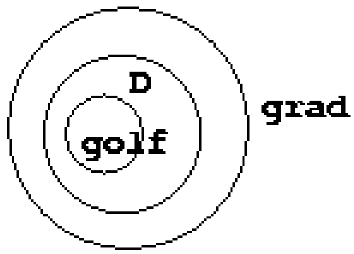

Figure 4: Counter example

1. $\forall x(f(x) \Rightarrow m(x))$
2. $\exists x(s(x) \wedge f(x))$
$- -$
$\forall x(s(x) \Rightarrow m(x))$
The claim is False. See figure 4.
4.i)

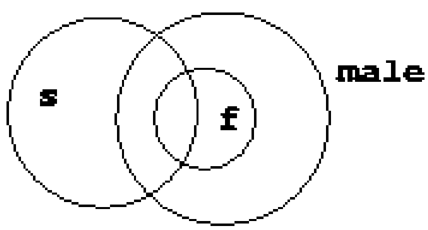

Figure 5: Counter example

1. $\forall x(f(x) \Rightarrow m(x))$
2. $\exists x(s(x) \wedge f(x))$
$- -$ $\exists x(s(x) \wedge m(x))$

Proof From 1, UI, $f(a) \Rightarrow m(a) - - - - - - - - - - 3$
From 2, EI, $s(a) \wedge f(a) - - - - - - - - 4$
From 4, simplification, $f(a) - - - - - - - - - - 5$

From 3,5, $m(a) - - - - - -6$
From 4, simplification, $s(a) - - - - - - -8$
From 6,8, $m(a) \wedge s(a) - - - - - 9$
From 9, EG, $\exists x(s(x) \wedge m(x))$