

CS603 Mathematical Concepts in Computer Science

Problem-Set-Linear Mapping-Automata Theory

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1 Linear Algebra

1. Find the $\text{span}\{(1,1)(1,-1)\}$.
2. Find the $\text{span}\{1, 1-t, (1-t)^2\}$.
3. $V = C, F = R$. Find the basis.
4. $V = M_{2 \times 2}(R)$, set of all real valued 2×2 matrices over real field. Find the basis.
5. $V = M_{2 \times 2}(C)$, set of all complex valued 2×2 matrices over real field. Find the basis.
6. Find the basis. $\{a_0 + a_2x^2 + \dots + a_{10}x^{10} \mid a_i \in R\}$.
7. Find the basis. $\left\{ \begin{bmatrix} a & x+iy \\ x-iy & b \end{bmatrix} \mid a, b, x, y \in R \right\}$.
8. Find the basis. $W_1 = \{(x, y, z) \mid x + 2y - z = 0\}$.
9. Find the basis. $W_2 = \{(x, y, z) \mid 2x - y - z = 0\}$.
10. Verify whether T is linear transformation. $T : R^2 \rightarrow R$ such that $T(x, y) = ax + by$ where $a, b \in R$. If T is a linear transformation verify rank-nullity theorem.
11. Verify whether T is linear transformation. $T : R^2 \rightarrow R^2$ such that $T(x, y) = (x - y, 0)$. If T is a linear transformation verify rank-nullity theorem.
12. Verify whether T is linear transformation. $T : R^3 \rightarrow R$ such that $T(x, y, z) = 2x - 3y + z$. If T is a linear transformation verify rank-nullity theorem.
13. Verify whether T is linear transformation. $T : R \rightarrow R$ such that $T(x) = 2x$. If T is a linear transformation verify rank-nullity theorem.

2 Automata Theory: Construct CFG

1. $\{a^m b^n \mid m \geq n, m, n \in N\}$
2. $\{a^m b^n \mid m \neq 3k + 1, k \in N\}$
3. $\{a^{2i} b^{2j} \mid i, j \geq 1\}$.
4. $\{(ab)^i c^{2j} \mid i, j \geq 1\}$.
5. $\{a^i b^j \mid i < j, i, j \geq 1\}$
6. $\{a^i b^j \mid i \leq j \leq 2i, i, j \geq 1\}$
7. $\{a^i b^j c^j \mid i, j \geq 1\}$
8. $\{a^i b^j c^k \mid i + j = k, i, j \geq 1\}$
9. $\{a^{2i} c b^{2j+1} \mid i, j \geq 1\}$
10. Set of strings $x \in \{a, b\}$ such that x contains twice as many a 's as b 's.
11. Set of palindromes of even length.
12. Set of palindromes of odd length.
13. $\{a^i b^j c^k \mid i = j \text{ or } i = k\}$
14. $\{a^i b^j c^k \mid i < j \text{ or } i > k\}$
15. $\{a^i b^j \mid i \leq 2j\}$.
16. $\{a^i b^j \mid i < 2j\}$.
17. $\{a^i b^j \mid i \neq j\}$.