B

INDIAN INSTITUTE OF TECHNOLOGY MADRAS

Tutorial - 3 (Prob. 1)

Department : Total No. of Pages

QUIZ - I QUIZ - II END SEMESTER MAKE - UP

STUDENT'S NAME ROLL NO :

B. TECH DD M.TECH MBA M.Sc M.S Ph.D

COURSE NO : PART :

QUESTION NO.	1	2	3	4	5	6	7	8	9	10
MARKS										

11	12	13	14	15	16	17	18	19	20	TOTAL

ANSWER ON BOTH SIDES OF THE PAPER
(Please use the space below for answering questions)

2 i) To prove,

$$P(A^c \cap (B \cup C)) = P(B) + P(C) - P(B \cap C) - P(C \cap A) - P(A \cap B) + P(A \cap B \cap C)$$

L.H.S.

$$P(A^c \cap (B \cup C)) = P(B \cup C) - P(A \cap (B \cup C)) \quad (\because P(A^c \cap B) = P(B) - P(A \cap B))$$

$$= P(B \cup C) - \left[P((A \cap B) \cup (A \cap C))\right] \quad (\text{Distributive Property})$$

$$= P(B \cup C) - \left[P(A \cap B) + P(A \cap C) - P(A \cap B \cap A \cap C)\right] \quad (\because P(A \cup B) = P(A) + P(B) - P(A \cap B))$$

$$= P(B \cup C) - \left[P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)\right]$$

$$= P(B) + P(C) - P(B \cap C) - P(A \cap B) + P(A \cap C) + P(A \cap B \cap C) = \text{R.H.S.} \quad \text{Proved}$$

Alternative Proof from R.H.S.

R.H.S

$$P(B) + P(C) - P(B \cap C) - P(C \cap A) - P(A \cap B) + P(A \cap B \cap C)$$

$$= P(A \cup B \cup C) - P(A)$$

$$= \cancel{P(A)} + P(B \cup C) - P(A \cap (B \cup C)) - \cancel{P(A)}$$

$$= P(D) - P(A \cap D) \quad [\text{Let, } B \cup C = D]$$

$$= P(A^c \cap D)$$

$$= P(A^c \cap (B \cup C)) \quad [\text{As we assumed } B \cup C = D]$$

$$= \text{L.H.S. Hence Proved.}$$

ii) Let A = set of no.s divisible by 7.

Let B the set of no.s divisible by 3.

Let C be the set of no.s divisible by 5.

$P(\bar{A} \cap (B \cup C)) = ?$

$P(B) = 200,\ P(C) = 120,\ P(B \cap C) = 40,\ P(C \cap A) = 17,$

$P(A \cap B) = 28,\ P(A \cap B \cap C) = 5.$

$P(\bar{A} \cap (B \cup C)) = P(B) + P(C) - P(B \cap C) - P(C \cap A) - P(C \cap A)$
$+ P(A \cap B \cap C)$

$= 200 + 120 - 40 - 17 - 28 + 5$

$= 240.$

$B = 200,\ C = 120,\ B \cap C = 40,\ C \cap A = 17,\ A \cap B = 28,\ A \cap B \cap C = 5$

$\bar{A} \cap (B \cup C) = B + C - B \cap C - C \cap A - A \cap B + (A \cap B \cap C)$

$= 200 + 120 - 40 - 17 - 28 + 5$

$= 240.$

3) Let $B_1, \ldots, B_n$ be partitions in sample space.
$A, B_1, \ldots, B_n$ be disjoint sets.

Then Bayes Theorem states that

$$P(B_i | A) = \frac{P(A|B_i)\, P(B_i)}{\sum_{j=1}^{n} P(A|B_j)\, P(B_j)}$$

$P(A|B_i)$ is the probability of A given B_i

$P(B_i|A)$ is the probability of B_i given A.

$\sum_{j=1}^{n} P(A|B_j) P(B_j)$ is the total probability of A.

1. i) $p = p(D/T) = \dfrac{P(T/D) \times P(D)}{P(T/D) \times P(D) + P(T/\neg D) \times P(\neg D)} = \dfrac{0.96 \times 0.01}{0.96 \times 0.01 + 0.06 \times 0.99}$

$= 0.139$. Ans

ii) $p' = 1 - (1-p) \times (1-p)$

$= 1 - (1-p)^2$

$= 1 - (0.861)^2$

$= 0.259$ Ans

5 i) $A \cap \bar{B} \cap \bar{C}$ ii) $A \cap B \cap C$ iii) $A \cup B \cup C$ iv) $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

v) $\overline{A \cup B \cup C} \equiv \bar{A} \cap \bar{B} \cap \bar{C}$ vi) $U - A \cap B \cap C \equiv \overline{A \cap B \cap C}$

4) $$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} p(A_i) \cancel{- \sum_{i=1}^{n} p(A_i)} - \sum_{i_1 < i_2}^{n} P(A_{i_1} \cap A_{i_2}) + \sum_{i_1<i_2<i_3}^{n} P(A_{i_1} \cap A_{i_2} \cap A_{i_3})$$

$$- + \ldots\ldots + (-1)^{n-1} \sum_{i_1<i_2<\ldots i_n}^{n} \left(\bigcap_{i=1}^{n} A_i\right)$$

Proof by Mathematical Induction.

<u>Base Case</u>

For $n=2$,

$$p(A_1 \cup A_2) = p(A_1) + p(A_2) - p(A_1 \cap A_2)$$

(from the formula $p(A \cup B) = p(A) + p(B) - p(A \cap B)$)

from expression for $n=2$, ~~$p(A_1)+p(A_2)$~~

$$p(A_1 \cup A_2) = p(A_1) + p(A_2) - p(A_1 \cap A_2)$$

Thus it is valid for base case.

<u>Induction Hypothesis</u>

We assume it to be true for n

<u>Inductive step</u>

To prove it to be true for $n+1$.

$$p(A_1 \cup A_2 \cup \ldots \cup A_{n+1}) = p\left(\bigcup_{i=1}^{n} A_i\right) + p(A_{n+1}) - P\left(\bigcup_{i=1}^{n} A_i \cap A_{n+1}\right) \rightarrow (i)$$

(from $p(A \cup B) = p(A) + p(B) - p(A \cap B)$)

Using Induction Hypothesis, on eq (i) we get,

$$p(A_1 \cup A_2 \cup \ldots \cup A_{n+1}) = \sum_{i=1}^{n} P(A_i) - \sum_{i_1<i_2} P(A_{i_1} \cap A_{i_2}) + - \ldots$$

$$+ (-1)^{n-1} P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_n}) + P(A_{n+1}) - P\left(\bigcup_{i=1}^{n} A_i \cap A_{i_{n+1}}\right)$$

$$= \sum_{i=1}^{n+1} P(A_i) - \sum_{i_1<i_2} P(A_{i_1} \cap A_{i_2}) + - \ldots$$

$$+ (-1)^{n-1} P(A_1 \cap \ldots \cap A_n) + (-1)^{n-1+1} P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_{n+1}})$$

$$= \sum_{i=1}^{n+1} P(A_i) - \sum_{i_1<i_2} P(A_{i_1} \cap A_{i_2}) + - \ldots$$

$$+ (-1)^{n} P(A_{i_1} \cap \ldots \cap A_{i_{n+1}})$$

$\therefore$ The expression holds for $n+1$.

$\therefore$ Proved by Mathematical Induction.