Assignment-1 - Solutions.

1) $\forall x \exists y \ (x+y=0)$, $\exists x \forall y (x+y=y)$, $\forall x \forall y (x \cdot y = y \cdot x)$, $\exists x \exists y (x+y=$

2) $(30+35-100)-(15+15+20)+5 = 120$

3) $L(x)$: x is a Lion $A(x)$: x is an animal $H(x,y)$: x is a head of

For example, "x is a head of a lion" in logic $\exists y (L(y) \wedge H(x,y))$

Given: $\forall x \ (L(x) \Rightarrow A(x))$

$$\overline{\forall x \ (\exists y \ (L(y) \wedge H(x,y)) \Rightarrow \exists y \ (A(y) \wedge H(x,y)))}$$

To prove,

Consider $\forall x \ (L(x) \Rightarrow A(x))$, $\neg \forall x \ (\exists y \ (L(y) \wedge H(x,y)) \Rightarrow \exists y \ (A(y) \wedge H(x,y))$

negation of the consequent.

we need to show that above expression is unsatisfiable (False).

$\neg \forall x \ (\exists y \ (L(y) \wedge H(x,y)) \Rightarrow \exists y \ (A(y) \wedge H(x,y)))$

$= \exists x \ \neg \ (\quad '' \quad)$

$= \exists x \ (\exists y \ (L(y) \wedge H(x,y)) \wedge \neg \exists y (A(y) \wedge H(x,y)))$

E.I $\exists y \ (L(y) \wedge H(a,y)) \wedge \neg \exists y \ (A(y) \wedge H(a,y))$

E.I $L(b) \wedge H(a,b) \wedge \neg \exists y \ (A(y) \wedge H(a,y))$

$L(b) \wedge H(a,b) \wedge \forall y \ (\neg A(y) \vee \neg H(a,y))$

UI $L(b) \wedge H(a,b) \wedge (\neg A(b) \vee \neg H(a,b))$

$(L(b) \wedge H(a,b) \wedge \neg A(b)) \vee (L(b) \wedge H(a,b) \wedge \neg H(a,b))$

$(L(b) \wedge H(a,b) \wedge \neg A(b)) \vee \text{False}$

$(L(b) \wedge H(a,b) \wedge \neg A(b)) \wedge \forall x \ (L(x) \Rightarrow A(x))$

UI $(L(b) \wedge H(a,b) \wedge \neg A(b)) \wedge (L(b) \Rightarrow A(b))$

$L(b) \wedge (L(b) \Rightarrow A(b)) \wedge H(a,b) \wedge \neg A(b)$

$A(b) \wedge H(a,b) \wedge \neg A(b) = \text{False}.$

this shows the validity of the argument.

4) B killed E.