Reducing Owl Entailment to Description Logic Satisfiability

Presented By – Kanthi Kiran S

Indian Institute of Technology, Madras

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Structure of the paper

- OWL FULL, OWL DL, OWL LITE
- Motivation for OWL reasoning
- Syntax and Semantics of OWL DL
- Syntax and Semantics of $SHOIN^+$
- Reduction from OWL DL to $SHOIN^+$
Motivation for OWL reasoning

- Reasoning with OWL is important in the Semantic Web if applications are to exploit the semantics of Ontology based metadata annotations.
- Highly optimized implementations of Description Logics exist.
- Subversions of OWL like OWL DL are very close to DLs.
- OWL’s RDF like syntax uses frame like constructs that do not correspond to DL axioms.
- OWL inference is defined in terms of ontology entailment rather than ontology satisfiability.
Class Constructors

- Exhaustive Enumeration of individuals
- Property Restriction
- Intersection of 2 or more classes
- Union of 2 or more classes
- Complement of a class.
Enumeration

```xml
<owl:Class>
  <owl:oneOf rdf:parseType="Collection">
    <owl:Thing rdf:about="#Eurasia"/>
    <owl:Thing rdf:about="#Africa"/>
    <owl:Thing rdf:about="#NorthAmerica"/>
    <owl:Thing rdf:about="#SouthAmerica"/>
    <owl:Thing rdf:about="#Australia"/>
    <owl:Thing rdf:about="#Antarctica"/>
  </owl:oneOf>
</owl:Class>
```
Property Restriction:

Property Restriction is of two types:

- value constraint
- cardinality constraint
AllValuesFrom

<owl:Restriction>
    <owl:onProperty rdf:resource="#hasParent" />
    <owl:allValuesFrom rdf:resource="#Human" />
</owl:Restriction>

someValuesFrom

<owl:Restriction>
    <owl:onProperty rdf:resource="#hasParent" />
    <owl:someValuesFrom rdf:resource="#Physician" />
</owl:Restriction>

Max-Cardinality, cardinality, min-cardinality

<owl:Restriction>
    <owl:onProperty rdf:resource="#hasParent" />
    <owl:maxCardinality rdf:datatype="&xsd;nonNegativeInteger">2</owl:maxCardinality>
</owl:Restriction>
Intersection

```xml
<owl:Class>
  <owl:intersectionOf rdf:parseType="Collection">
    <owl:Class>
      <owl:oneOf rdf:parseType="Collection">
        <owl:Thing rdf:about="#Tosca" />
        <owl:Thing rdf:about="#Salome" />
      </owl:oneOf>
    </owl:Class>
  </owl:intersectionOf>
</owl:Class>

<owl:Class>
  <owl:oneOf rdf:parseType="Collection">
    <owl:Thing rdf:about="#Turandot" />
    <owl:Thing rdf:about="#Tosca" />
  </owl:oneOf>
</owl:Class>
</owl:Class>
```
Union

<owl:Class>
  <owl:unionOf rdf:parseType="Collection">
    <owl:Class>
      <owl:oneOf rdf:parseType="Collection">
        <owl:Thing rdf:about="#Tosca" />
        <owl:Thing rdf:about="#Salome" />
      </owl:oneOf>
    </owl:Class>
    <owl:Class>
      <owl:oneOf rdf:parseType="Collection">
        <owl:Thing rdf:about="#Turandot" />
        <owl:Thing rdf:about="#Tosca" />
      </owl:oneOf>
    </owl:Class>
  </owl:unionOf>
</owl:Class>
</owl:unionOf>
</owl:Class>
- Class Axioms
- Property Axioms
- Facts
Class Axioms

- DisjointClass($d_1, d_2, ..$)
- EquivalentClass($d_1, d_2, ..$)
- SubClassOf($d_1, d_2$)
SubClassOf

<owl:Class rdf:ID="Opera">
  <rdfs:subClassOf rdf:resource="#MusicalWork" />
</owl:Class>

EquivalentClass

<owl:Class rdf:about="#US_President">
  <equivalentClass rdf:resource="#PrincipalResidentOfWhiteHouse" />
</owl:Class>

DisjointClass

<owl:Class rdf:about="Man">
  <owl:disjointWith rdf:resource="#Woman" />
</owl:Class>
Property Axioms

- Object Property
- Data Property
Property Axioms

- EquivalentProperty
- SubProperty
- InverseOf
SubProperty

<owl:ObjectProperty rdf:ID="hasMother">
   <rdfs:subPropertyOf rdf:resource="#hasParent"/>
</owl:ObjectProperty>

InverseOf

<owl:ObjectProperty rdf:ID="hasChild">
   <owl:inverseOf rdf:resource="#hasParent"/>
</owl:ObjectProperty>
## SHOIN syntax and semantics

<table>
<thead>
<tr>
<th>Constructor Name</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>atomic concept</td>
<td>A</td>
<td>$A^I \subseteq \Delta^I$</td>
</tr>
<tr>
<td>datatypes</td>
<td>D</td>
<td>$D^I \subseteq \Delta^D$</td>
</tr>
<tr>
<td>abstract role</td>
<td>R</td>
<td>$R^I \subseteq \Delta^I \times \Delta^I$</td>
</tr>
<tr>
<td>datatype role</td>
<td>U</td>
<td>$U^I \subseteq \Delta^I \times \text{time} \Delta^D$</td>
</tr>
<tr>
<td>Individuals</td>
<td>o</td>
<td>$o^I \in \Delta^I$</td>
</tr>
<tr>
<td>data values</td>
<td>v</td>
<td>$v^I = v^D$</td>
</tr>
<tr>
<td>inverse role</td>
<td>$R^-$</td>
<td>$(R^-)^I = (R^I)^-$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>----------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td><strong>conjunction</strong></td>
<td>$C_1 \cap C_2$</td>
<td>$(C_1 \cap C_2)' = (C_1' \cap C_2')$</td>
</tr>
<tr>
<td><strong>disjunction</strong></td>
<td>$C_1 \cup C_2$</td>
<td>$(C_1 \cup C_2)' = (C_1' \cup C_2')$</td>
</tr>
<tr>
<td><strong>negation</strong></td>
<td>$\neg C$</td>
<td>$(\neg C)' = \Delta' - C'$</td>
</tr>
</tbody>
</table>
### Axiom Name

<table>
<thead>
<tr>
<th>Axiom Name</th>
<th>Syntax</th>
<th>Semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept Inclusion</td>
<td>$C_1 \subset C_2$</td>
<td>$C'_1 \subset C'_2$</td>
</tr>
<tr>
<td>individual Inclusion</td>
<td>$a : C$</td>
<td>$a'^I \in C'_I$</td>
</tr>
<tr>
<td>individual Equality</td>
<td>$a = b$</td>
<td>$a'^I = b'^I$</td>
</tr>
<tr>
<td>individual Inequality</td>
<td>$a \neq b$</td>
<td>$a'^I \neq b'^I$</td>
</tr>
<tr>
<td>Concept Existence</td>
<td>$\exists C$</td>
<td>$(C'_I) \geq 1$</td>
</tr>
</tbody>
</table>
From OWL DL to SHOIN

- From a previous Paper by Decker, there is a translation of an OWL class to Description Logic concept.
- If C is a class the corresponding Description Logic Concept is $V(C)$.
- Using this we can easily convert all the OWL axioms into DL axioms.
Theorem

The Translation from OWL DL to SHOIN$^+$ preserves equivalence.